Ch 1 – Fluid Mechanics

1.1 Density

$P = | F / A |$ or
$P\, dA = dF$

SI units of Pressure is the Pascal
1 Pascal = 1 N / m^2

Density, ρ
(The Greek, r, rho)

$\rho = m / V$

Some common densities		
material	**cgs**	**mks**
Water	1	1,000
Ice	0.917	917
Air	0.00129	1.29
Helium (He)	0.00018	0.179
Copper (Cu)	8.96	8,960
Gold (Au)	19.3	19,300
Platinum (Pt)	21.4	21,400
Iridium (Ir)	22.4	22,400
Aluminum	2.7	2,700
Lead (Pb)	11.34	11,340
Mercury (Hg)	13.6	13,600

Important Note:
If you notice, air (and other gasses) are approximately 1000 times less dense than liquids and solids

That means the spacing between molecules is 10 times greater in all three dimensions
(l×w×h) → (10x10x10=1000)

Example using density

The spring constant of the pressure gauge is 1000 N/m, and the piston has a diameter of 2.00 cm. As the gauge is lowered into water, what change in depth causes the piston to move in by 0.500 cm?

$P = F/A$; $F = PA$ where $P = \rho gh$

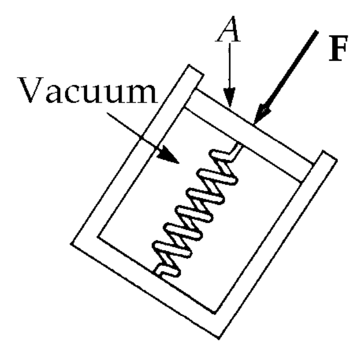
$F = -kx$; and $F = \rho ghA$

$k \quad x \quad = \rho\, g\, h\, A$

$h = k \quad x \quad / \quad \rho \quad g \quad A$

$h = 1000*0.005 / (1000(9.8)\pi 0.01^2)$

$h = 1.62$ meters



1.2 Pressure

Pascal's Law

A change in pressure to a fluid is transmitted undiminished to every point of the fluid and to the wall of the container See bottom of page

$P_1 = P_2$

$F_1 / A_1 = F_2 / A_2$

$F_1 A_2 = F_2 A_1$

Pressure below a surface must include atmospheric pressure, P_o, as well pressure attributable to the liquid above, P_{liq}

$F_{weight} = m \ g$

$\rho = m/V; \ m = \rho V$

$F_w = \rho \ V \ g$

$F_w = \rho (Ah) g$

$F_w = \rho \ A \ h \ g$

$F_w / A = \rho \ A \ h \ g / A$

$P_{liq} = \rho \ g \ h$

h = depth of liquid

$P = P_o + P_{liq}$

$P_o = 1.013 \times 10^5$ Pa

Pliq is also called Gauge

Application

Pyramids of Egypt

$P_1 = P_2$

$F_1 / A_1 = F_2 / A_2$

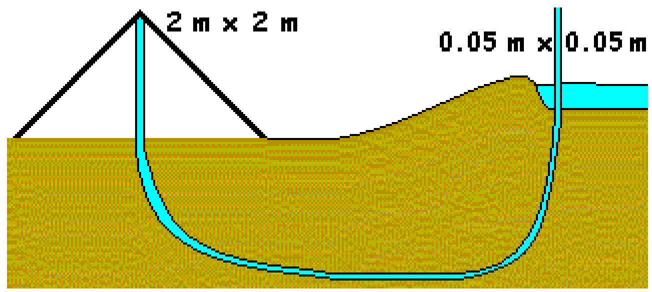
$\rho_{stone} \ h_{stone} \ g = \rho_{H20} \ h \ g$

$2500 \ (2) \ g = 1000 \ h \ g$

$h = 5$ meters

So the water column must remain 5 meters higher than the stone block

Tubes have been found below the pyramids of Egypt to a central chamber, possibly leading to a dried lake.
If this is the case, one method to raise large stone blocks [(2x2x2) meters] is application of Pascal's Law



Example: Force on a dam with depth

$\Delta F = P \ \Delta A$

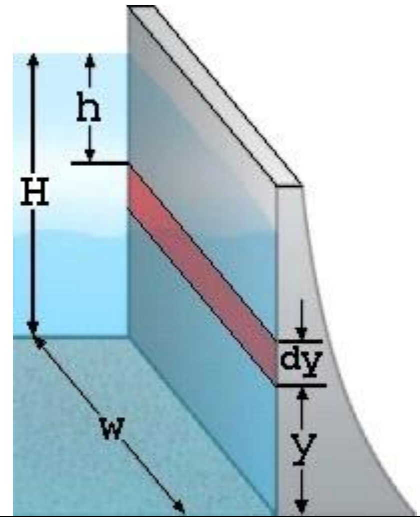
$\Delta F = \rho \ g \ (H-y) \ w \Delta y$ (use Calculus)

$F = \frac{1}{2} \rho g w H^2$

What is the force on a 80 meter wide dam which has a depth of 30 meters?

$F = \frac{1}{2} \ \rho \ g \ w \ H^2$

$F = \frac{1}{2}(1000)10(80)30^2$



Pressure

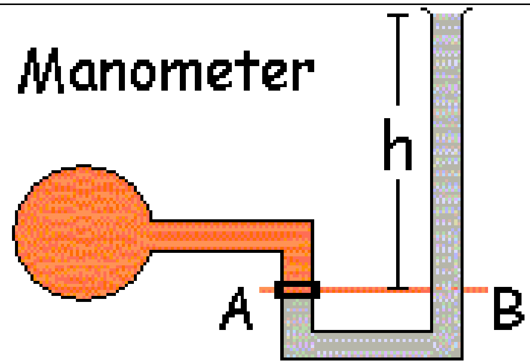
F = 360 MN or the same weight as 15,000 SUVs

Pressure readings referenced in every day usage generally is commonly referred to as gauge pressure.

(i.e. 32 pounds / in^2 in our car tires. This is 32 lb/in^2 beyond atmospheric pressure)

Absolute pressure (P) is atmospheric pressure (P_o) plus gauge pressure (ρ g h)

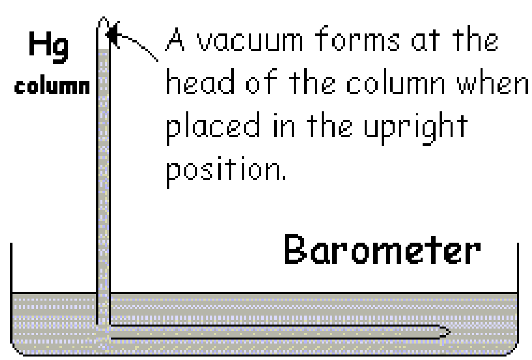
$P = P_o + \rho g h$



The key to the manometer is that the place of attachment, point A, is at the same level of the beginning measurement of h.

Points A & B must be at same pressure, (at same level), so ρgh is the gauge pressure.

Common units of pressure	Compared to P_o (1 atm)
Atmosphere	1 atm
Hg column	760 mm
Torr	760 torr
Pascal	1.013×10^5 Pa
Bar	1.013 bar
Mbar	1013 mbar



If the outside pressure goes higher … the surface of the Hg is pushed down even more…so the Hg in the column is pushed up in the column more.

Vise versa if the outside pressure goes down.

lb/in^2	14.7 lb/in^2	
The above Mercury column experiment is a very powerful demo. But we don't perform this demo because Hg is a hazardous chemical. So we instead we decide to use water. Is this feasible in our classroom?		$P + \rho g h = P_o + \rho g h$ $0 + \rho g h = P_o + 0$ h is 0 meters above the surface $\rho g h = P_o$ $h = P_o / \rho g$ $h = 1.013\times10^5 / (1000)(9.8)$ $h = 10.3$ meters

1.3 Static Equilibrium in Fluids: Pressure and Depths

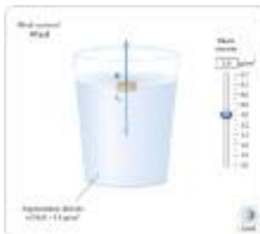
Covered above and below in 1.6

1.4 Archimedes' Principle and Buoyant Forces

The buoyant force on an object is equal to the weight of the displace fluid

Everyone should cover this subject during lab class. After all labs are complete, we will review this subject. Common topics usually covered during lab class are	The volume of the displace fluid is V If the displace fluid is water then the weight of the displaced water is $m\,g = (\rho_{H2O}V)\,g \rightarrow$ so $F_B = (\rho_{H2O}V)\,g$ So if a crown displaces 30 cc's of water (cc = cm^3, also cc = ml and 1 cc of water has the mass of 1 gram)

- Bermuda triangle vanishing ships
- Underwater weight lifting using gold and aluminum weight sets
- Hot air balloons
- Archimedes and the King with the golden crown
- And more



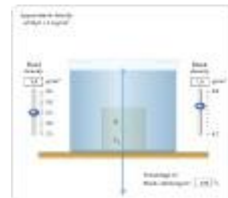
$F_B = (\ \rho_{H2O} \ * \ V\ )\ \ g$

$F_B = (1\ g/cc * 30cc)\ 1000\ cm/s^2$

(I usually use $g = 10\ m/s^2$ or $1000\ cm/s^2$ during lecture)

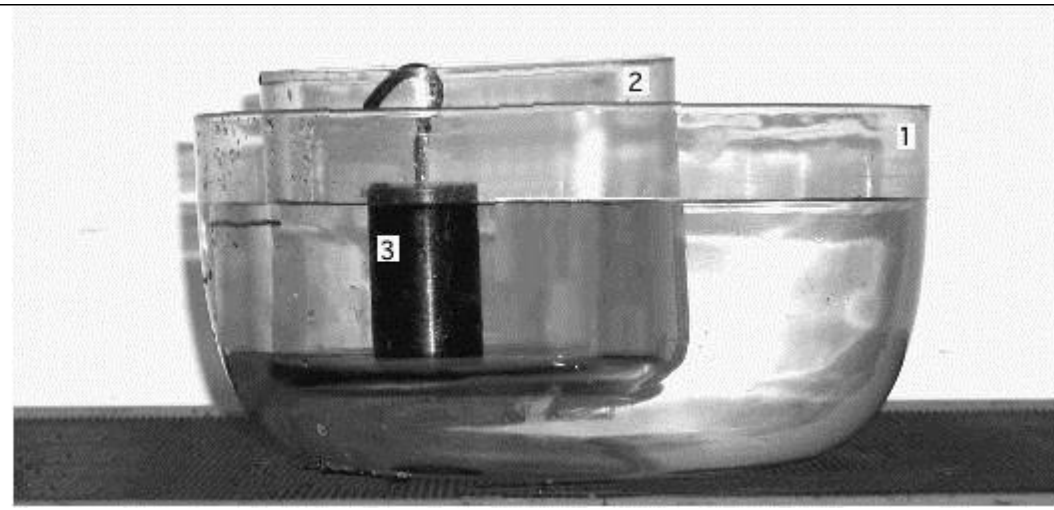
$F_B = 30{,}000$ dynes **or**

$F_B = 0.3$ Newtons (100,000 dynes = 1 Newton)

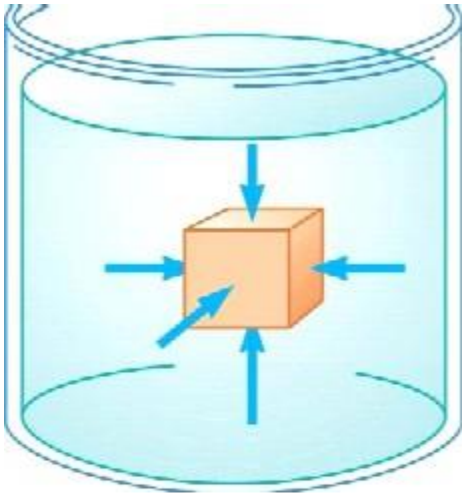


Active Figure 14.10

[Archimedes Principle: Buoyancy]()



1.5 Application of Archimedes' Principle



Buoyancy force…just sum up vectors…

You see that there is a F_{Net} up.

1.6 Fluid Flow and Continuity

There are two main types of fluid flow.

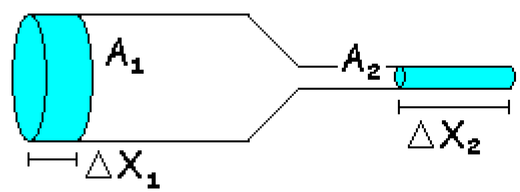
- Steady (laminar)
- Turbulent

Viscosity measures the internal friction of a fluid (i.e. Elmer's glue has more internal friction than water)

To be able to model real fluids we must make a few assumptions (which usually are

Equations

$V_{cyclinder} = A * x$; $\rho = m / V$; $\bar{v} = \frac{\Delta x}{\Delta t}$



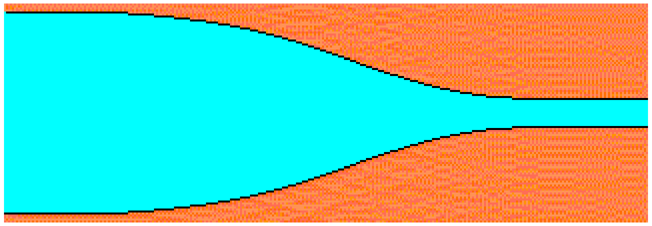
completely true).

- The fluid is non-viscous
- The fluid is steady
- The fluid is incompressible
- The fluid has no rotation

A wide river with a flow rate of 10,000 gallons per minute (gpm) suddenly narrows. What occurs?

ans: the water flows at a much faster velocity

Most everyone already knew this answer from watching TV, movies, or other personal experience.



Why? The 10,000 gpm still must flow at the same rate to make it through the gorge. If the 10,000 gpm doesn't flow at 10,000 gpm, it backs up and forms a lake behind the gorge.

Just for fun… the mass of 1 gallon of water is approximately 4 kg. So what is the mass of

$V_1 = A_1 * x_1$ $V_2 = A_2 * x_2$

$\rho = m_1 / V_1$ $\rho = m_2 / V_2$

$\rho = m_1 / A_1 x_1$ $\rho = m_2 / A_2 x_2$

$\Delta x_1 = v_1 \Delta t$ $\Delta x_2 = v_2 \Delta t$

$\rho = m_1 / A_1 v_1 t_1$ $\rho = m_2 / A_2 v_2 t_2$

Since density of water is the same in both equations, we can set the two equations equal to each other

$m_1 / A_1 v_1 t_1 = m_2 / A_2 v_2 t_2$

$A_2 v_2 \ (m_1 / t_1) = A_1 v_1 \ (m_2 / t_2)$

← Now let's look at the example to the left. The mass per unit time was constant in either the wide part of the river or when the river flowed through the gorge.

(It was 40,000 kg / min)

Since $m/\Delta t$ is constant $m_1/\Delta t = m_2/\Delta t$ and the equation simplified down to

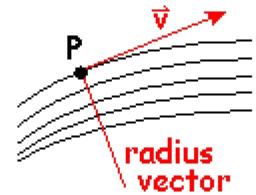
$$A_1 v_1 = A_2 v_2$$

water flowing out of the gorge?

10,000 gal/min (4 kg/gal) = 40,000 kg/min

<u>One last note on a particle in Laminar flow.</u>

The path taken by a fluid under steady flow is called a stream line (block lines), these form curves (parts of multiple circles). Thus the velocity vector is tangent to each one of these stream lines. I'm showing one velocity vector at point P.



Example:
The channel at the Aquaduct at Segovia (Spain) has a channel that measures 1.5 wide by 1.8 tall measured in meters. Immediately before the aquaduct is broken by a severe earthquake (fictional event) the water was flowing at 10cm/sec while being 1/3 full, what is the diameter of water stream when it reaches the ground 28 meters below?

Water volume per unit time is

1.5m (1/3*1.8 m) (0.1 m/s)

$V/t = 0.09\ m^3/sec$

Or Area (velocity)

1.5m (1/3*1.8 m) (0.1 m/s)

$A\ v = 0.09\ m^3/sec$

$V / t = A_f\ v_f$

$0.09 = A_f\ (23.6)$

$A_f = 0.0038\ m^2$

$\pi r^2 = 0.0038$

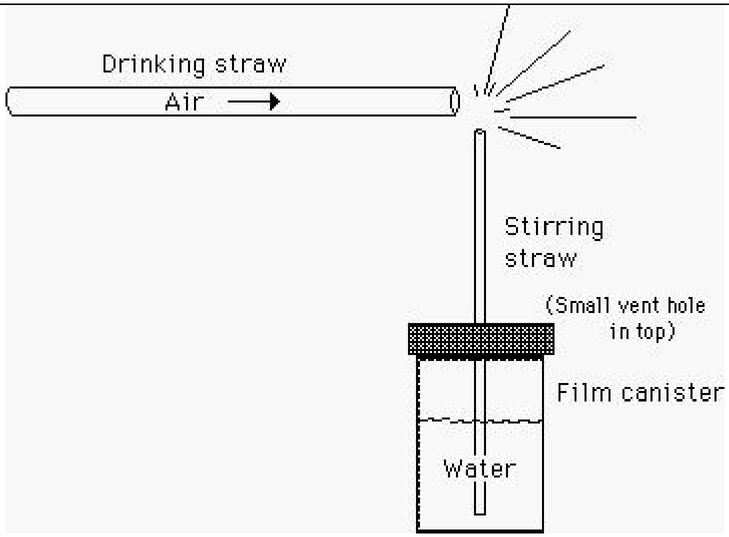
r = 3.5 cm

Energy of position transfers to energy of position

$\frac{1}{2}mv^2 = mgh$

$v^2 = 2(10m/s^2)\ 28m$

$v_{bottom} = 23.6\ m/s$

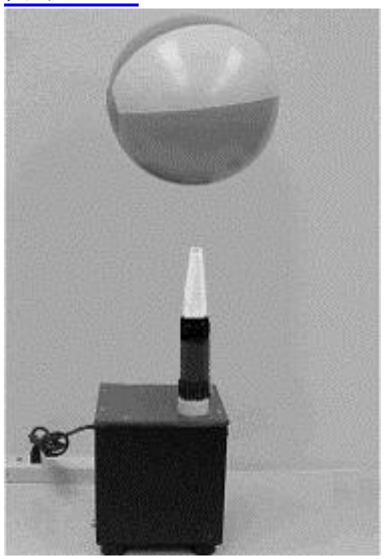


[Bernoulli's Atomizer FM-C-BA]

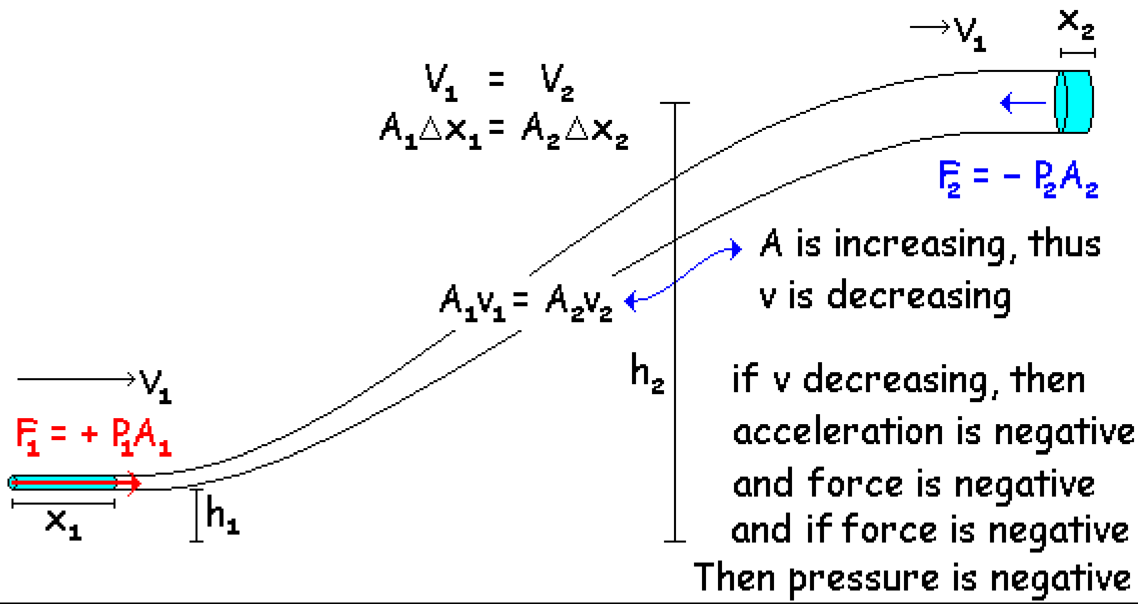
1.7 Bernoulli's Equations

Bernoulli's Beach Ball

FM-C-BB



Derivation: $P_1 - P_2$



It's already been established that pressure is measured in N / m^2; and volume is measured in m^3

So what are the units of PV (as in PV = nRT)?

$N / m^2 * m^3 = Nm$

(units of Work and energy)

So just from the units we expect: Work = $\Delta P V$

Let's look at this from another view point

Work = $F \Delta x$

Work = $F \Delta x (A / A)$ multiplying by 1

Work = $(F/A) (\Delta x A)$

So if pressure is changed, you do work

Work = $P_1V + (-P_2V)$ & Work = ΔK

But what if the pipe slopes up or down. Gravity will also do work on the system.

Work = $\Delta K + \Delta U$

$P_1V - P_2V = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgh_2 - mgh_1$

$P_1V + \frac{1}{2}mv_1^2 + mgh_1 = P_2V + \frac{1}{2}mv_2^2 + mgh_2$

So final equals initial…or no changes…so

$PV + \frac{1}{2}mv^2 + mgh = \text{constant}$

Divide through by volume

Work = Δ P V ...very similar to above.

So we can conclude that a change in pressure for a constant volume is the amount of work done on a system.

$P + \frac{1}{2}(m/V)v^2 + (m/V)gh = \text{constant}$

$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$

1.8 Applications of Bernoulli's Equation

A Venturi tube may be used as a fluid flow meter. If the difference in pressure is $P_1 - P_2 = 21.0$ kPa, find the fluid flow rate in cubic meters per second, given that the radius of the outlet tube is 1.00 cm, the radius of the inlet tube is 2.00 cm, and the fluid is gasoline ($\rho_{gas} = 700$ kg/m^3).

$A_{in} = \pi(2)^2;$

$A_{out} = \pi(1)^2$

$A_{in}v_{in} = A_{out}v_{out}$

$4\ v_{in} = 1\ v_{out}$

$v_{out} = 4v_{in}$

$P + \frac{1}{2}\rho v^2 + \rho g h = P_f + \frac{1}{2}\rho v_f^2 + \rho g h_f$

$P_{in} + \frac{1}{2}\rho v_{in}^2 = P_{out} + \frac{1}{2}\rho v_{out}^2$

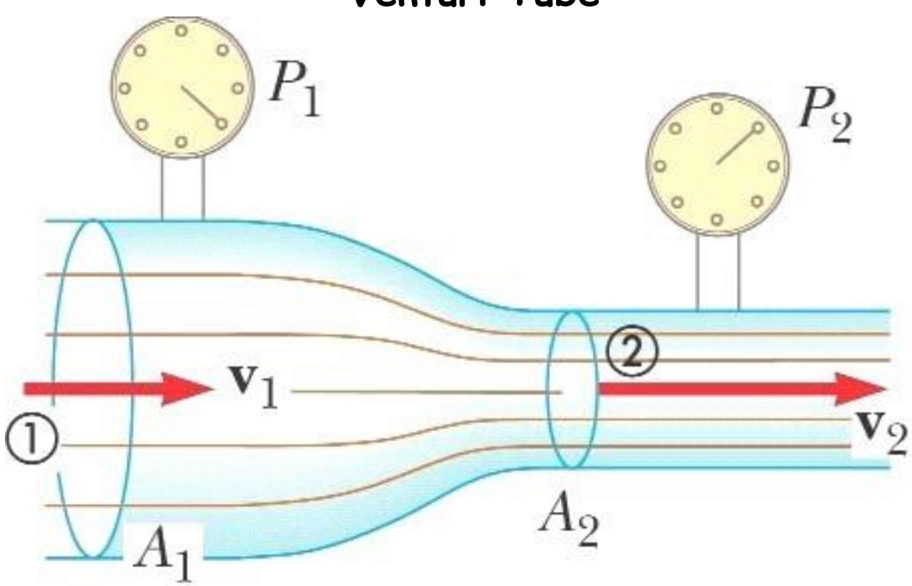
$P_{in} - P_{out} + \frac{1}{2}\rho v_{in}^2 = \frac{1}{2}\rho\, v_{out}^2$

$21{,}000 + \frac{1}{2}\rho v_{in}^2 = \frac{1}{2}\rho(4v_{in})^2$

$15v_{in}^2 = (2/\rho)\ 21{,}000$

$v_{in} = 2$ m/s

Venturi Tube

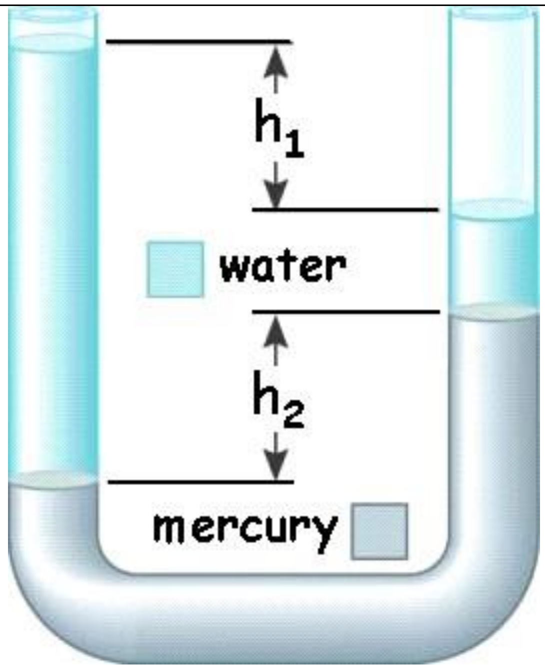


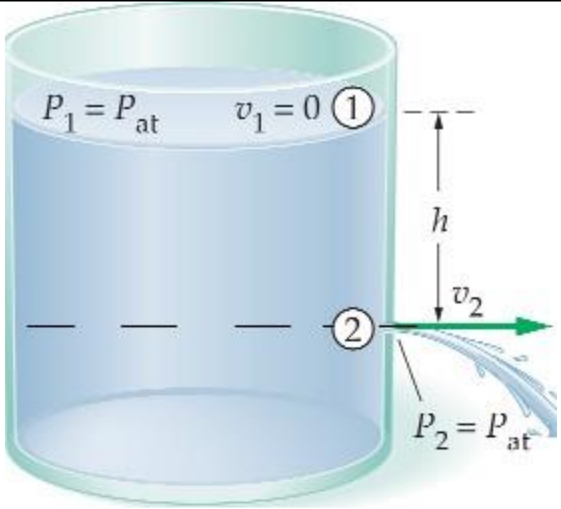
Another example to the right...

So what is h_2?

Hint:

What is the weight of the two columns?





What is v_2?

$P + \frac{1}{2}\rho v^2 + \rho gh = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$

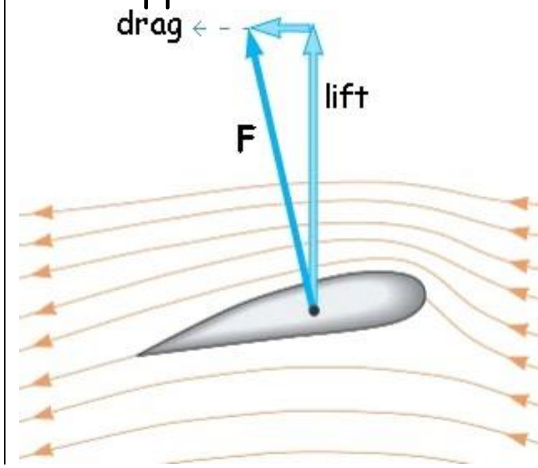
$P_{at} + 0 + \rho gh = P_{at} + \frac{1}{2}\rho v_2^2 + 0$

$\rho gh = \frac{1}{2}\rho v_2^2$

$v_2^2 = 2gh$

This results is Torricelli's Law

Last Application…



1.9 Viscosity and Surface Tension

We have ignored internal friction (like usual), but how would our discussion change if we didn't?

As with surface friction, friction that fluid experiences is also ALWAYS opposed to its flow (come speak to me if you don't agree with "always", if you don't believe this, it comes from a misinterpretation of the definition of a system)

$P_1 - P_2 = 8\pi\eta \, v \, L / A$; where eta is the coefficient of viscosity W

Viscosity was originally measured in poise where water at 20°C was 0.01 poise (dyne sec/cm^2). We use SI units (see table).

1 poise = 0.1 Ns/m^2

Multiply both sides of the above equation and solve for vA

$vA = (P_1 - P_2)A^2 / 8\pi\eta L$ (remember V/t = vA)

TABLE 15–3
Viscosities (η) of Various Fluids ($N \cdot s/m^2$)

Honey	10
Glycerine (20 °C)	1.50
10-wt motor oil (30 °C)	0.250
Whole blood (37 °C)	2.72×10^{-3}
Water (0 °C)	1.79×10^{-3}
Water (20 °C)	1.0055×10^{-3}
Water (100 °C)	2.82×10^{-4}
Air (20 °C)	1.82×10^{-5}